INTERPHASE MASS AND HEAT TRANSFER IN A COMPACT DISPERSE SYSTEM

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Limiting relations are derived for the Sherwood number, which characterizes convective diffusion to a solid sphere, a droplet, or a bubble in a constricted stream, applicable to low values of the Reynolds number and high values of the Peclet number.

Although the problem of heat or mass transfer to isolated particles in a stream of fluid has been thoroughly analyzed over various ranges of the Reynolds number Re and the Peclet number Pe (see, for instance, the survey of theoretical studies in [1] and experimental studies in [2]), the solution of an analogous problem which involves solid particles, droplets, or bubbles in a dense cloud is more difficult, because the flow field which becomes constricted around a single particle and which affects the heat convection or the diffusion appreciably is usually unknown. The only exception here is the practically important case where $\mathrm{Re} < 1$ and $\mathrm{Pe} \gg 1$, when this field can be roughly estimated on the basis of the semiempirical cell model of constricted flow, and for which case the diffusion or the heat conduction have been analyzed in the "thin diffusion layer" approximation [1, 3-5].

The results based on this cell model are of doubtful validity, however, foremost on account of the indeterminacy of the boundary conditions which must be stipulated at the outer surface of a cell around every particle so that the hydrodynamic problem of a stream around a particle can be solved. For instance, various well-known variants of the cell model (Happel, Kuvabary, et al.) lead to substantially different conclusions concerning the velocity and pressure field around a particle as well as the heat and mass flow toward it. For this reason, the problem of convective diffusion toward solid particles in a compact disperse system was analyzed in [6] on the basis of the more rigorous flow model which had been proposed in [7]. Here we will extend the results obtained in [6] to compact systems of droplets or bubbles, the constricted flow of a carrier fluid in such systems having been analyzed in [8]. We will use here the method of reducing the equation of convective diffusion to a certain equation of heat conduction with a variable thermal conductivity, such an equation having been already solved in [9] (see also [10]). Furthermore, the results based on this theory will be compared with test data pertaining to compact systems of solid particles. In order to be specific, we will discuss convective diffusion and the Sherwood number but, in view of the well-known analogy between diffusion and heat conduction, all results will apply also to heat conduction and the Nusselt number.

According to [9, 10], the equation of convective diffusion in the "thin diffusion boundary layer" approximation at the interphase boundary reduces to the following approximate equation:

$$\frac{\partial c}{\partial x} = D\frac{\partial}{\partial \psi}\left(v_\theta \frac{\partial c}{\partial \psi}\right), \tag{1}$$

with the tangential velocity v_θ of the fluid at that boundary assumed to be expressible in terms of the flow function ψ. If the interphase boundary is spherical, then the flow function in its vicinity can be represented as

$$\psi \approx -(U_1 a\xi + 3/4U_2\xi^2)\sin^2\theta, \tag{2}$$

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where in the special case of an isolated particle the coefficients U_1 and U_2 are equal, respectively, to the fluid velocity at the equator of such a particle ($\theta = 1/2\pi$) and the relative velocity of the fluid far away from this particle.

In the case of constricted flow around a solid sphere, a droplet, or a bubble with surfaces stagnated by surfactant materials, the first term in (2) vanishes and coefficient U_2 becomes [7, 8]:

$$U_2 = \varphi(\rho)\,U,\quad \varphi(\rho) = \frac{1}{2-3\rho}\left\{\left[18\rho\left(1-\frac{3}{2}\rho\right)+\frac{81}{4}\rho^2\right]^{1/2}+\frac{9}{2}\rho\right\}+1 \tag{3}$$

(for simplicity, we consider a monodisperse system of particles with a radius a).

The second extreme case is realized in a stream around droplets or bubbles with a viscosity μ and without surfactants, when the second term in (2) may be disregarded as negligibly smaller than the first term. In this case the results of [8] yield

$$U_1 = \frac{1}{2}\vartheta(\rho,\ \varkappa)\,U,\quad \vartheta(\rho,\ \varkappa) = \frac{1+\zeta}{1+1/3\zeta+\varkappa},\quad \varkappa = \mu'/\mu, \tag{4}$$

where $\zeta = \zeta(\rho, \varkappa)$ is the positive root of the equation

$$\zeta^3 + 3(1+\varkappa)\zeta^2 - \frac{9(2+3\varkappa)}{2-3\rho}\zeta - \frac{9(2+3\varkappa)}{2-3\rho} = 0. \tag{5}$$

It is not difficult to show that, when $0 < \rho < 2/3$, this equation has only one positive root which becomes zero when $\rho = 0$. Curves representing the relation between ζ and ρ are shown in Fig. 1 for various values of $\varkappa$.

Solving Eq. (1) under the conditions

$$c = c_0\,(r \to \infty;\ r = a,\ \theta = 0),\quad c = 0\,(r = a,\ \theta \neq 0)$$

accurately, by the same method which was used in [9, 10], will yield the following earlier expression [6] for the mass current toward a solid sphere

$$J = 7.98\,(D^2a^4U_2)^{1/3}\,c_0, \tag{6}$$

and for the mass current toward a droplet or a bubble

$$J = 5.72\,(Da^3U_1)^{1/2}\,c_0. \tag{7}$$

Introducing the Sherwood number S, using also (3) and (4), we obtain:

for a solid sphere (or a droplet with a stagnant surface)

$$S = B\mathrm{Pe}^{1/3},\quad B = 0.998\varphi^{1/3}(\rho), \tag{8}$$

and for a droplet or a bubble with a free surface

$$S = C\mathrm{Pe}^{1/2},\quad C = 0.462\vartheta^{1/2}(\rho,\ \varkappa). \tag{9}$$

We will note certain limitations on the applicability of the critical relations (8) and (9). First of all, they apply to monodisperse systems. Extending them to polydisperse systems leads to trivial results, it essentially amounts to replacing functions $\varphi(\rho)$ and $\vartheta(\rho, \varkappa)$ in (4) and (5) by other functions for the determination of coefficients B and C in (8) and (9), functions which additionally depend also on the moments of the size spectrum of particles and which follow directly from the analysis in [7, 8] concerning the hydrodynamics of constricted flow around particles. Such a generalization of formula (8) can be found in [6].

Furthermore, it has really been assumed in the derivation of Eq. (9) that the flow function around a bubble or a droplet can be closely enough approximated by the first term in Eq. (2). Obviously, this is valid only so long as the diffusion current (6) remains much smaller than the diffusion current (7). Considering that coefficients U_1 and U_2 are of the same order of magnitude in the case of a stream around a droplet or a bubble, we obtain by a simple transformation the following applicability criterion for formula (9):

$$\mathrm{Pe}^{1/3} \gg \varkappa. \tag{10}$$

In the general case of an arbitrary relation between Pe and $\varkappa$, when inequality (10) may not necessarily hold true, the method of [9, 10] is not suitable here. However, one may use then the more roughly approximate polynomial method, which had been first proposed in [11, 12] for the analysis of convective

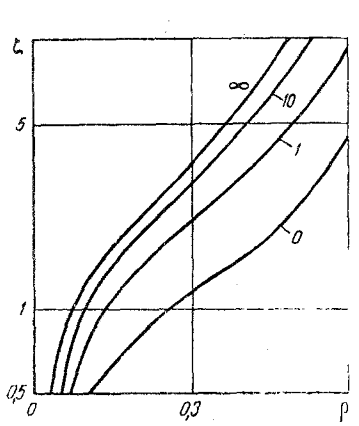

Fig. 1

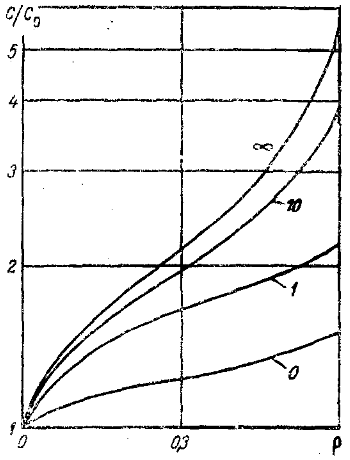

Fig. 2

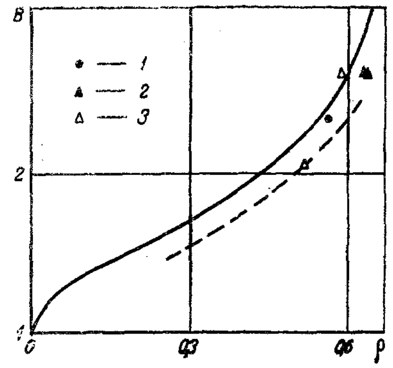

Fig. 3

Fig. 1. Positive root ζ of Eq. (5), as a function of the volume concentration ρ, for various values of $\varkappa$ (indicated by the numbers next to the curves).

Fig. 2. Ratio C/C_0 as a function of the volume concentration ρ, for various values of $\varkappa$ (indicated by the numbers next to the curves).

Fig. 3. Coefficient B as a function of the volume concentration ρ: the solid curve represents formula (8); the dashed curve represents $B \sim (1-\rho)^{-1}$ [16]; 1) data from [13]; 2) data from [14]; 3) data according to MacConnell and described in [15].

diffusion, and derive by it an interpolation formula for S of the same kind as had been considered in [1, 4]. It is to be noted that relation (9) applies to pure systems containing no surfactants. This relation can be extended to systems which contain surfactants, by an analogy to the analysis in [1, 5] based on the general theory which has been developed in [9] concerning the effect of such additives on the mobility of the interphase boundary.

When $\rho \to 0$, the filtration velocity U of a fluid passing through a cluster of particles approaches the Stokes velocity U_0 of a single particle, while functions $\varphi(\rho)$ and $\vartheta(\rho, \varkappa)$ become, respectively, equal to 1 and $(1+\varkappa)$. As a result, coefficients B and C become $B = B_0 = 0.998$ and $C = C_0 = 0.462\,(1+\varkappa)^{-1/2}$, while (8) and (9) become the well-known formulas describing the mass transfer which involves a single particle and the ambient medium (see, for instance, [9]). The ratio C/C_0, which characterizes the change in the Sherwood number for single droplets or bubbles, is shown in Fig. 2 (the change in the Sherwood number is not shown here, because the definition of the Peclet number is based on the fluid filtration velocity in the case of a compact cluster but on the Stokes stream velocity in the case of single particles).

A comparison between the relations in Fig. 2c and those obtained by mass- and heat-transfer tests in disperse systems of the liquid–gas kind is difficult to make, because the surfactant content is usually either not checked at all or not reported in the data. Considering the strong effect which stagnancy of a droplet or bubble surface may have on the process of mass and heat transfer, a direct comparison between theory and available test data pertaining to such systems seems premature. It is feasible, however, to verify the theory by a comparison with the test results of many experiments regarding the mechanism of mass and heat transfer in close-packed and fluidized beds of granular solid particles.

Most test data of this kind are represented either graphically or in the form of empirical equations for the so-called Colburn factor

$$j = S\mathrm{Re}^{-1}\mathrm{Sc}^{-1/3} = j\,(\mathrm{Re},\ \varepsilon) \tag{11}$$

as a function of the Reynolds number Re or some effective Reynolds number including not only Re but also the porosity $\varepsilon = 1-\rho$, used in different ways by different authors. The theoretical expression for the Colburn factor in (11), corresponding to the critical relation (8), is

$$j = B\mathrm{Re}^{-2/3} = \mathrm{Re}_*^{-2/3},\ \mathrm{Re}_* = \mathrm{Re}B^{-3/2}. \tag{12}$$

It is this expression which ought to be compared with formulas of the (11) kind.

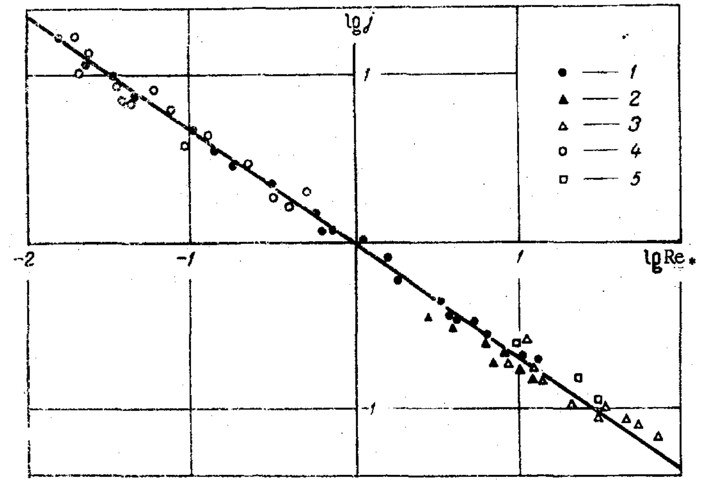

Fig. 4. Theoretical relation (12) between the Colburn factor j and the modified Reynolds number Re_*, and test data: 1) from [13]; 2) from [14]; 3) from [15]; 4) from [18]; 5) from [19].

First of all, we will compare test data with the formula for coefficient B, in (3) and (8), as a function of ρ (Fig. 3). Tests were usually performed with the bed porosity varying from 0.3 to 0.5, unfortunately, making it impossible to verify (8) for $B = B(\rho)$ over the entire range of ρ (or ε). However, the arithmetic mean values calculated from the data in [13-15] and shown in Fig. 3 agree closely enough with the theoretical curve. Empirical relations for the Sherwood number S or the Colburn factor j as functions of the porosity ε have been proposed by several authors. According to [16], for instance, $S \sim \varepsilon^{-1}$ (dashed line in Fig. 3). Other authors ([17], for instance) have proposed a relation $S \sim \varepsilon^{-2}$ applicable to systems with a high Reynolds number (Re = 100-1000) and, therefore, not suitable for comparison with this theory. A survey of other such empirical relations can be found in [2].

The relation (12) for the Colburn factor j as a function of the modified Reynolds number Re_*, when plotted to a logarithmic scale, appears as a straight line (Fig. 4). In order to compare it with test data, the latter are expressed in terms of relation (11) and then referred to j, Re_* coordinates, with Re_* determined from (12). In Fig. 4 are also shown, after necessary conversions, test results from [13, 18] applicable to relatively low values of the Reynolds number Re and test results from [14, 15, 19] applicable to high values of the Reynolds number. According to Fig. 4, the relation $j \sim \mathrm{Re}^{-2/3}$ (or the equivalent relation $S \sim \mathrm{Re}^{1/3}$) is valid when the Reynolds number remains smaller than unity, but in reality is valued up to $\mathrm{Re}_* \sim 100$. This has been noted in [3] already and it fully confirms the viewpoint stated in [6] that results pertaining to low values of the Reynolds number in a constricted stream should apply also to high values of Re, inasmuch as the separation of the boundary layer is in this case impeded.

In conclusion, we note that relations (8) and (9) can be applied to heat convection if the Sherwood number and the Schmidt number (in the definition of the Peclet diffusion number) are replaced by the Nusselt number and the Prandtl number, respectively.

NOTATION

a	is the radius of a particle;
B	is the coefficient in formula (8);
C	is the coefficient in formula (9);
c	is the concentration of material;
D	is the diffusivity;
J	is the total diffusion current toward the surface of a particle;
j	is the Colburn factor;
k	is the integral coefficient of mass transfer;
$\mathrm{Pe} = 2a\mathrm{U}/\mathrm{D}$	is the Peclet number;
$\mathrm{Re} = 2a\mathrm{U}/\nu$	is the Reynolds number;
Re_*	is the modified Reynolds number in formula (12);
$\mathrm{S} = 2a\mathrm{k}/\mathrm{D}$	is the Sherwood number;
$\mathrm{Sc} = \nu/\mathrm{D}$	is the Schmidt number;

U	is the filtration velocity;
U_1, U_2	are the coefficients in formula (2);
v_θ	is the tangential velocity of a fluid at the surface of a particle;
ε	is the porosity;
ζ	is the positive root of Eq. (5);
θ	is the angular coordinate;
ϑ	is the function defined in (4);
$\varkappa = \mu'/\mu$;	
μ	is the dynamic viscosity of the carrier medium;
μ'	is the dynamic viscosity of the particle material;
ν	is the kinematic viscosity of the ambient medium;
ξ	is the volume concentration of solid particles;
φ	is the function defined in (3);
ψ	is the flow function.

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